Robust Zero-crossing Detection in Noisy signals Using Persistent Homology

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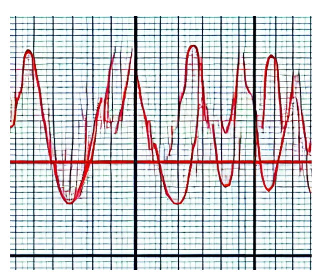
Abstract

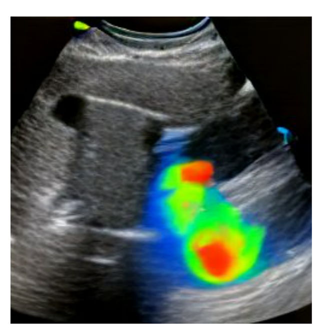
we explore a novel application of zero-dimensional persistent homology from Topological Data Analysis (TDA) for bracketing zero-crossings of both one-dimensional continuous functions, and uniformly sampled time series. We present an algorithm and show its robustness in the presence of noise for a range of sampling frequencies. In comparison to state-of-the-art software-based methods for finding zeros of a time series, our method generally converges faster, provides higher accuracy, and is capable of finding all the roots in a given interval instead of converging only to one of them. We also present and compare options for automatically setting the persistence threshold parameter that influences the accurate bracketing of the roots.

Motivation

The determination of zero-crossings in a discrete signal is of immense interest in various fields for applications, such as:

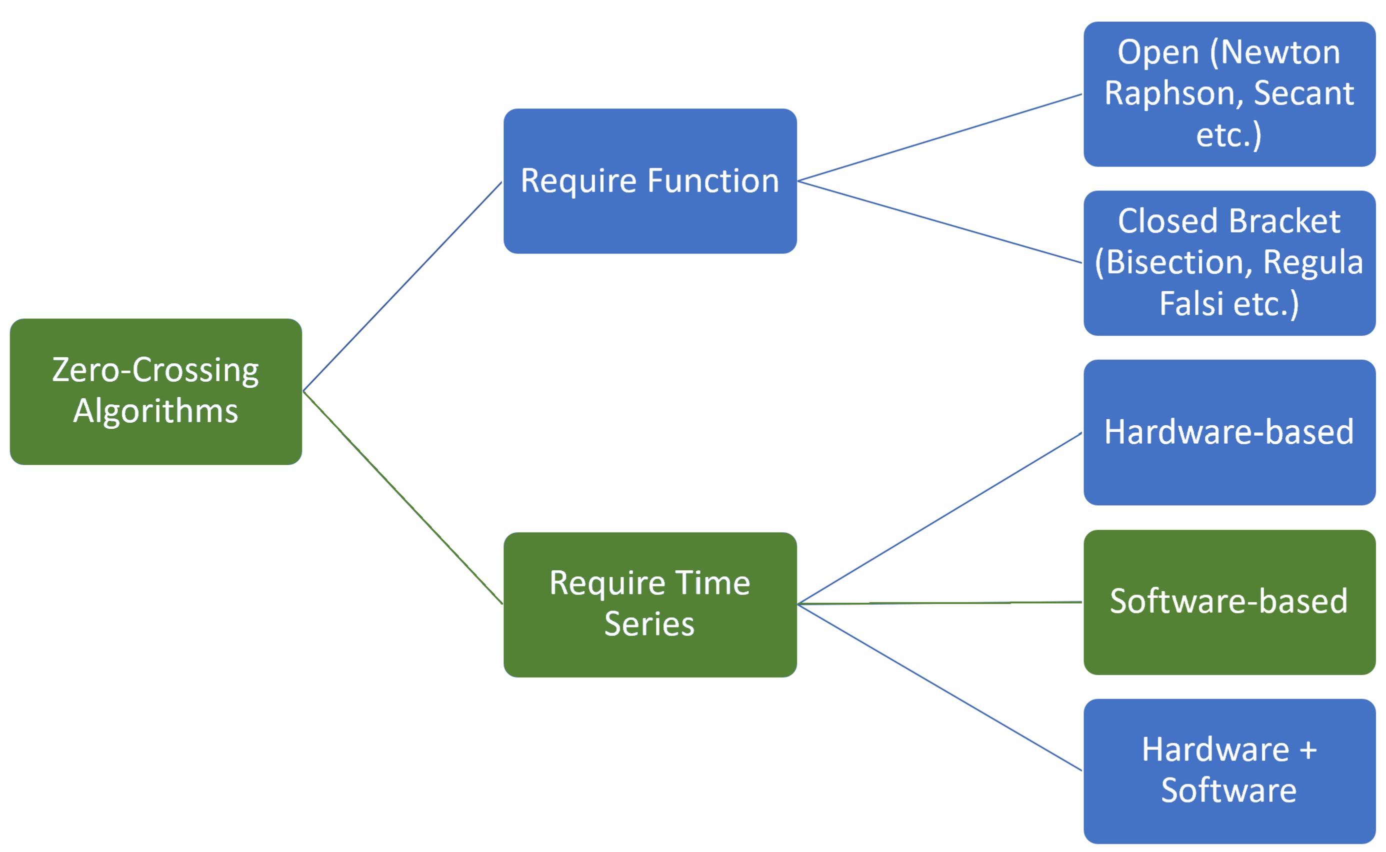
- Frequency Determination
- Motor Fault Detection
- Strain Elastography





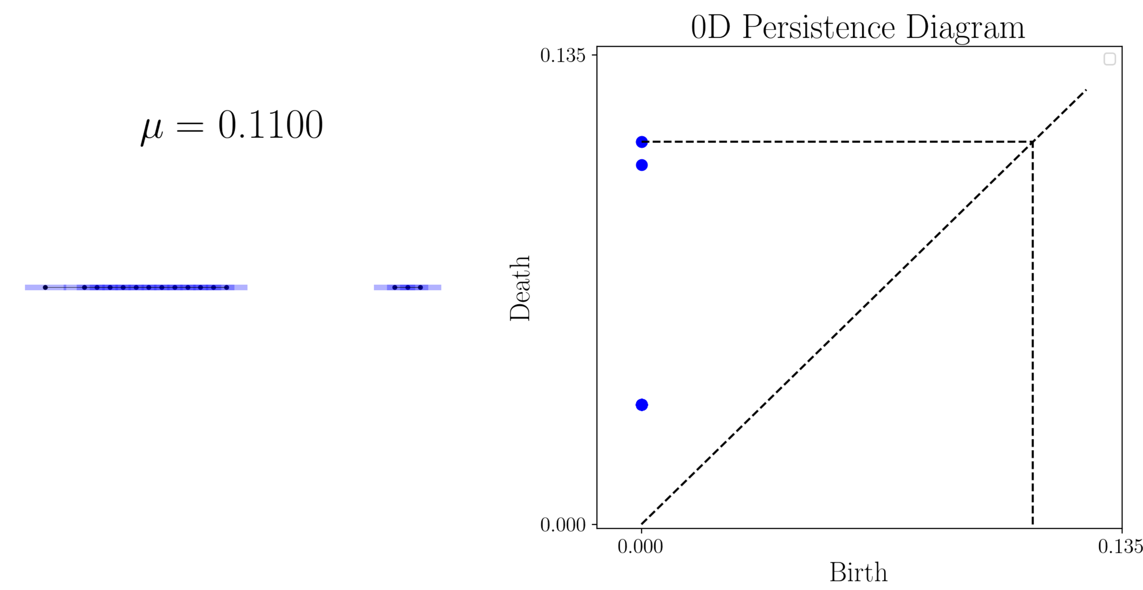
Available Methods

Methods for detecting zero-crossings are primarily divided into two types: (a) which require a function, (b) which require a time series. Both these types have further classifications. The proposed method is applicable to both functions and time series, but is primarily for discrete 1-dimensional data.



Point Cloud Persistent Homology

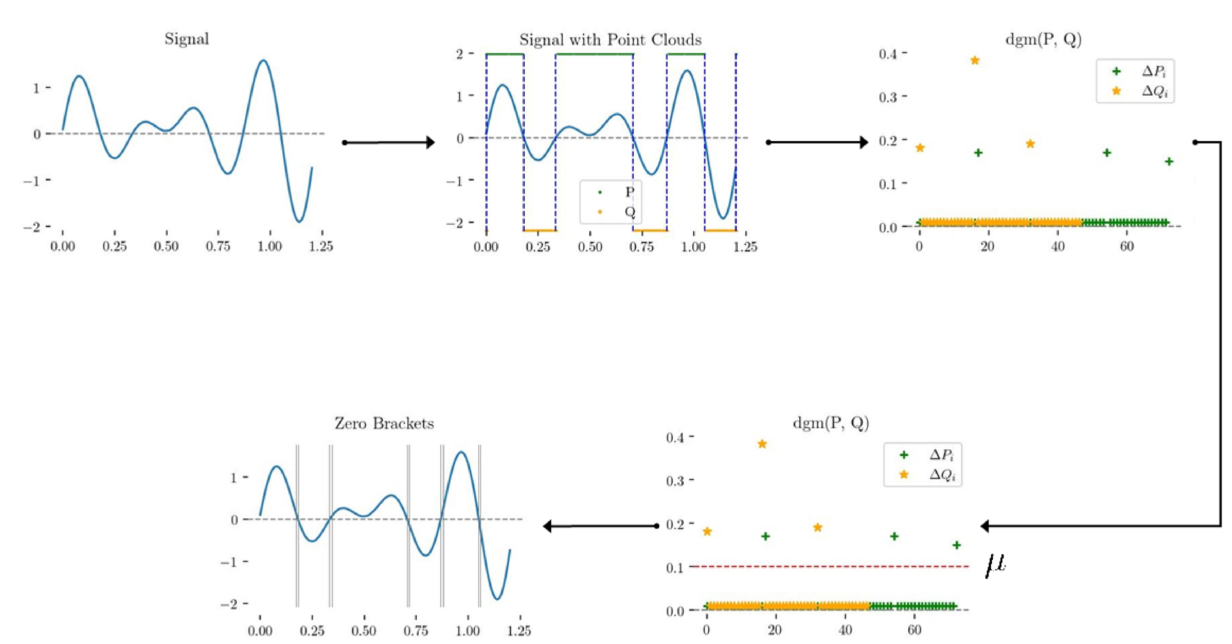
Persistent homology captures information about the shape of a parameterized space by tracking how its homology changes as the parameter varies. For 1-dimensional point clouds (points on a line), that parameter is an interval of length μ centred at each point. Here, the number of clusters are being measured.



1. Take a point cloud $\chi = \{x_1, \cdots, x_n\} \subset \mathbb{R}$ given as input.
2. For a fixed $\mu \geq 0$, the Rips complex is given by
$$R(\chi, \mu) = \{\sigma = \{x_0, \cdots, x_d\} \mid \|x_i - x_j\| \leq \mu \text{ for all } i, j\}.$$
3. Raise the μ from $0 \to \infty$, and construct a sublevelset filtration, along with a persistence diagram accordingly.

Method

1. Binarize the point cloud based on sign of function value.
2. Compute 0D persistence for each point cloud.
3. Fix a persistence threshold μ and take points above it in a sorted persistence diagram.
4. Find brackets using the index of the points.



Setting a Threshold μ

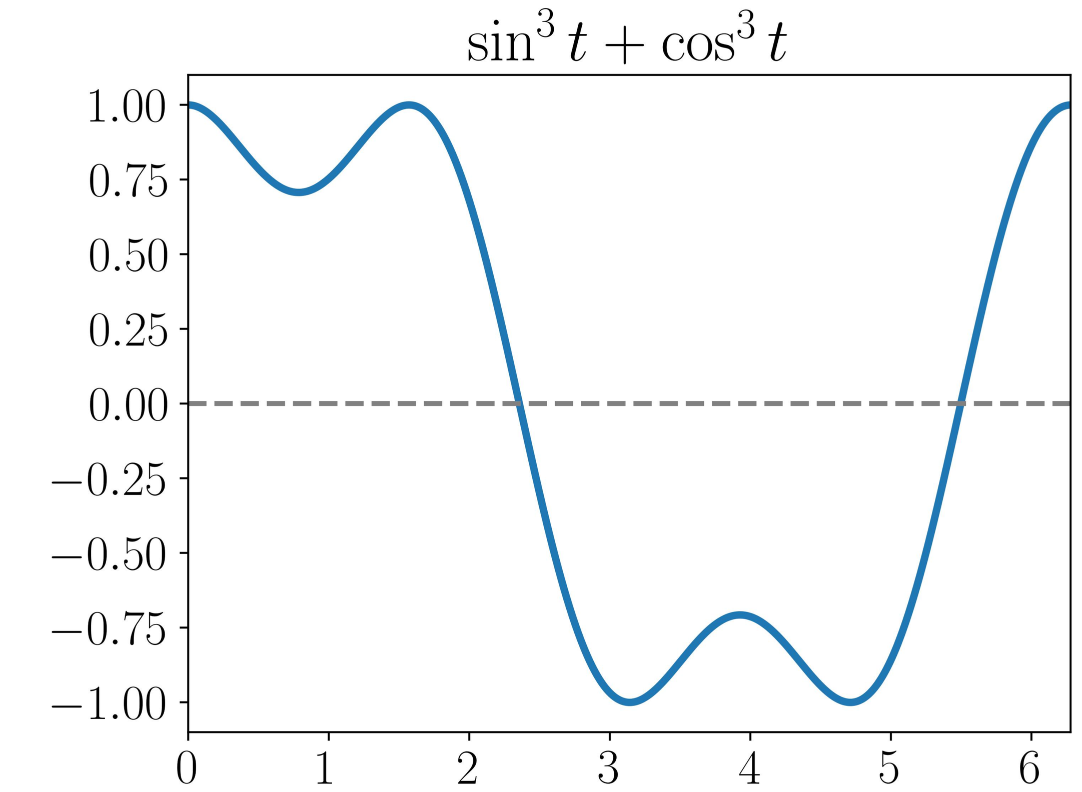
Automatic Computation

1. Univariate outlier detection method such z-score of 3.
2. Machine Learning outlier detection method such as Isolation Forest.

Manual Computation

1. Plotting the persistence diagram and finding points with higher persistence OR plotting a histogram of points against persistence and choosing a threshold based on low count.

Comparison with State-of-the-art [1]



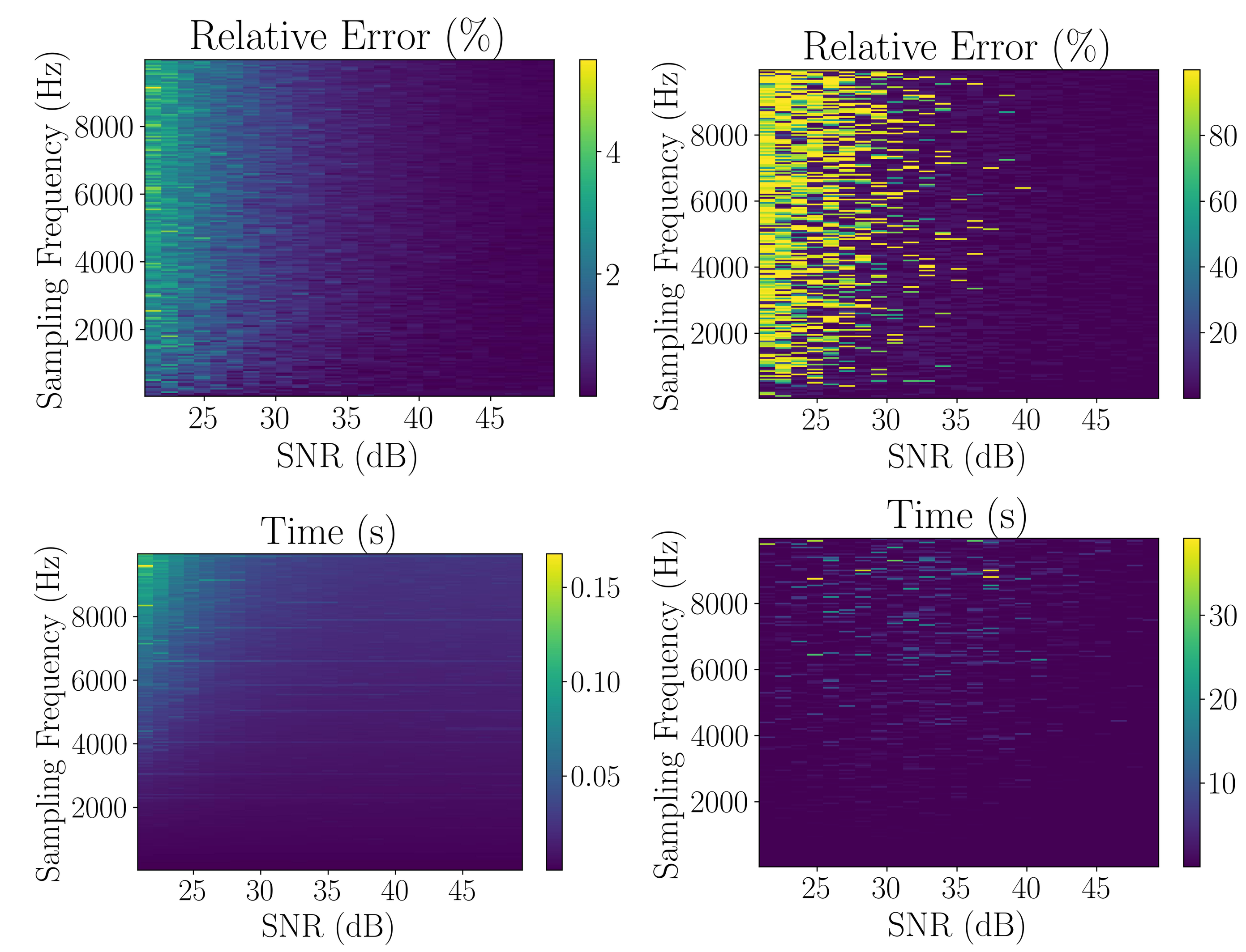


Figure 1. Relative erros and time for computation taken by our algorithm (left column) and Molinaro's algorithm [1] (right column)

Conclusion

1. A novel tool for detecting ALL zero crossings in a discrete-time signal in an interval.
2. Faster and more accurate than the state-of-the-art algorithm.
3. Tested for robustness against noise as low as 15dB.

References

[1] A. Molinaro and Ya.D. Sergeyev.
An efficient algorithm for the zero crossing detection in digitized measurement signal.
Measurement, 30(3):187–196, oct 2001.

https://github.com/TeaspoonTDA/teaspoon

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